Newton's Laws and Applications

Outline

- Force

- Newton's Laws
  
  Inertia
  Mass and Acceleration
  Action-Reaction

Force

- Why do objects move?
- Why can we observe some changes in the motion?
- How can an object start moving from rest?
- How can an object stop while moving?

Force can be the reason behind the changes which we observe in the motion of an object.

\[
\begin{align*}
\vec{F}_1 & \text{ slows down the object} \\
\vec{F}_2 & \text{ speeds up the object} \\
\vec{F}_3 & \text{ speeds up the object and it lifts it up as well (changes the direction)}
\end{align*}
\]
- The unit of the force is Newton (N)
- The force is a vector.

If two or more forces are applied to an object, then the total force is calculated by the vectoral addition.

\[ \vec{F}_{\text{net}} = \sum_i \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots \]

**Newton's First Law: Inertia**

Every object continues in its state of motion, or in other words, moves with a uniform velocity, as long as the net force acting on the object is zero.

\[ \vec{F}_{\text{net}} = \sum_i \vec{F}_i = 0 \]  
Balance: object keeps the state of its motion.

The changes are observed when \( \vec{F}_{\text{net}} = \sum_i \vec{F}_i \neq 0 \)

However, the object resists against the force not to change its motion. This resistance is called **Inertia**.

Heavier objects resist more than light objects.

\[
\text{Inertia} \sim \text{mass}
\]

Mass = Measurement of inertia (Inertia parameter)

\[ \text{M} > \text{m} \]
M will move slower than m under the same force.
Newton's Second Law

\[ \vec{F} = \vec{m} \text{ fast} \]

\[ \vec{F} = 2\vec{m} \text{ fast} \]

\[ \vec{F} = 3\vec{m} \text{ slowest} \]

\[ \vec{F} = \vec{m}, \Delta v \]

\[ \vec{F} = \vec{m}, \Delta v' \]

\( \Delta v \) and \( \Delta v' \) are not equal; thus, duration of experiments, \( \Delta t \) and \( \Delta t' \), are also important. If we check the correlation between \( \Delta v \) (\( \Delta v' \)) and \( \Delta t \) (\( \Delta t' \)), we observe that

\[ \frac{\Delta v}{\Delta t} = \frac{\Delta v'}{\Delta t'} \]

(say for the first block)

When an object is subjected to a force \( \vec{F} \), its velocity changes; however, the change in the velocity is proportional to duration of the experiment. If we consider the ratio between the velocity and the duration

\[ \frac{\Delta v}{\Delta t} \rightarrow \text{acceleration} = \text{constant} \]

The duration of experiment (\( \Delta t \)) is not important any more.
The acceleration of an object is directly proportional to the net force acting on the object, and inversely proportional to its mass.

\[ \vec{F}_{\text{net}} = \sum \vec{F}_i = m \vec{a} \to \text{Newton's Second Law} \]

**Newton's Third Law**

The objects in a physical contact apply forces to each other. These forces are of the same magnitude but in the opposite directions.

Newton's third law states that forces exist in pairs. If you generate a force, another force will be also created in the opposite direction with the same magnitude.

If we call one of these forces "**action**", the other force is called "**reaction**".
Special Forces

1-Gravitational Force
Two objects exert each other a force due to their masses. Each force has the same magnitudes, but they are in opposite directions:

\[ \vec{F}_{12} \rightarrow \text{force from } m_1 \text{ on } m_2 \]
\[ \vec{F}_{21} \rightarrow \text{force from } m_2 \text{ on } m_1 \]

This force is called gravitational force. The gravitational force between two objects is always attractive and it is given by Newton's equation as

\[ \left| \vec{F}_{12} \right| = \left| \vec{F}_{21} \right| = G \frac{m_1 m_2}{r^2} \]

- \( m_1, m_2 \rightarrow \text{masses} \)
- \( r \rightarrow \text{separation} \)
- \( G \rightarrow \text{Newton's constant} \)

\[ G = 6.67 \times 10^{-11} \text{ N m}^2 \text{kg}^{-2} \]

\[ \left[ G \right] = M^{-1} L^3 T^{-2} \]
The object also attracts the Earth. Why does the Earth not move? The answer can be given by checking the accelerations of the Earth and the object. Let us employ Newton's second Law:

For the Earth

\[ F = G \frac{M \cdot m}{R^2} = M \cdot a_E \]
\[ a_E = G \frac{m}{R^2} \]

For the object

\[ F = G \frac{M \cdot m}{R^2} = m \cdot a \]
\[ a = G \frac{M}{R^2} \]

\[ m \ll M \implies a \gg a_E \]
When we drop an object from the height $y$ from, it starts falling (Recall free falling), due to the gravitational force exerted by the Earth. Its acceleration is

$$a = G \frac{M}{R^2} = g \quad \text{gravitational acceleration}$$

in vector notation

$$\vec{a} = \vec{g} = (-9.81 \text{ m/s}^2) \hat{j}$$

Weight is nothing but the gravitational force on an object exerted by the Earth. It is always towards the center of the Earth.
If you squeeze or stretch a spring, you feel a force from the spring that is in the opposite direction of your move. If you squeeze or stretch more, the force will be stronger. Thus, the force depends on the displacement from the equilibrium point.

\[
\Delta \vec{x} = \vec{x} - \vec{x}_0 \quad \rightarrow \quad \text{displacement}
\]

\[
\vec{F} = -k \Delta \vec{x} \quad \rightarrow \quad \text{Hook's Law}
\]

\[
k \quad \rightarrow \quad \text{Spring constant (N/m)}
\]

The equilibrium of a spring is defined as to be the state in which the net force is zero.

Since displacement changes during the motion, the spring force and the acceleration will change. When the object comes to the equilibrium point the spring force becomes zero. However, since the object has a non-zero velocity, it starts squeezing the spring. After the object passes the equilibrium point, the spring force switches its direction and the objects slows down till it stops. At this point the spring force pushes the object back to equilibrium point. The motion repeats itself.
This is a complete circle of the motion. This motion is called the "Simple Harmonic Motion".

The figure shows a complete cycle of a simple harmonic motion.

Since the simple harmonic motion is a periodic motion; then, we can define its period, frequency and angular velocity.

\[ \omega = \sqrt{\frac{k}{m}} \quad \longrightarrow \quad \text{Angular velocity} \]

\[ T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}} \quad \longrightarrow \quad \text{Period} \]

\[ f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad \longrightarrow \quad \text{Frequency} \]
**Systems of Springs**

If the system is in equilibrium:

\[
\begin{align*}
\vec{F} &= \vec{F}_1 + \vec{F}_2 \\
F &= k \cdot \Delta x \\
F_1 &= k_1 \cdot \Delta x \\
F_2 &= k_2 \cdot \Delta x
\end{align*}
\]

\[
k = k_1 + k_2
\]

\(k \rightarrow\) total spring constant of the system

\[
\Delta x = \Delta x_1 + \Delta x_2
\]

\[
\Delta x = \frac{F}{k}, \quad \Delta x_1 = \frac{F}{k_1}, \quad \Delta x_2 = \frac{F}{k_2}
\]

\[
\frac{F}{k} = \frac{F}{k_1} + \frac{F}{k_2}
\]

\[
\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}
\]

\[
k = \frac{k_1 \cdot k_2}{k_1 + k_2}
\]

**3-Tension Force**

If two objects are also connected to each other with a rod, string etc., then the force on the objects generates some tension on the rod. This tension applies forces on the objects, and these forces are always pulling the objects.
### 4-Normal Force

Normal force is, indeed, a reaction force exerted on objects when they sit or move over some surfaces. Normal force is always perpendicular to the surface.

![Diagram of normal force](image)

### 4-Friction Force

If an object moves on a surface or in a medium with viscosity (air, water etc) the surface or the medium resists to the object's motion. This resistance is called "friction force"

![Diagram of friction force](image)

Friction force is always in the opposite direction of the motion. It slows down the object and even stops it, but it never starts or works for the motion.

The friction forces are not the same on the object when it moves or it is at rest.
The magnitude of the friction force is proportional to the magnitude of the normal force.

\[ f \sim N \]

\[ f_s = \mu_s N \quad \rightarrow \quad \text{Static friction force} \]

\[ f_k = \mu_k N \quad \rightarrow \quad \text{Kinetic friction force} \]
Forces in Circular Motion

By Newton's Second Law

\[ \vec{F} = m \vec{a} \]

If there is an acceleration, then there is a net force on the object.

In the circular motion the acceleration always points the center of the circular trajectory.

\[ \vec{a} \rightarrow \text{Centripetal acceleration (} a_c \text{)} \]

\[ \vec{F} = m \vec{a}_c \rightarrow \text{Centripetal force (} F_c \text{)} \]

If an object in a circular motion is being pulled toward the center, why does it not fall to the center?

Recall Newton's third law: Acton reaction forces

If the centripetal force does not make the object fall onto the center, there should be another force on the object, which cancels the centripetal force.

\[ \vec{F} \rightarrow \text{Centrifugal force} \]
Free Body Diagram (FBD)

Free body diagram (FBD) is the diagram which shows each force acting on an object.

If a system has objects more than one, FBD should be drawn individually for each object.

Solutions to the problems should start with the free body diagrams.
Applications of Newton’s Laws

Example 1

A block of mass 2.0 kg is held in balance on an incline of angle \( \theta = 60^\circ \) by a horizontal force. Determine the magnitude of the force.

Solution:

Since the object is in balance (equilibrium), the net force on the object has to be zero.

\[
\begin{align*}
\sum F_x &= 0 \\
F - F_{\text{normal}} \cos 30 &= 0 \\
N &= \frac{mg}{\sin 30} \\
F &= F_{\text{normal}} \cos 30 = \frac{mg}{\sin 30} \cos 30 \\
F &= \frac{mg \cos 30}{\sin 30} \\
F &= \frac{(2.0 \text{ kg})(9.81 \text{ m/s}^2) \cos 30}{\sin 30} \\
F &= 33.9 \text{ N}
\end{align*}
\]
Example 2

A block of mass 3.70 kg on a frictionless inclined plane with $\theta=30^\circ$ is connected by a cord over a massless, frictionless pulley to a second block of mass 2.30 kg hanging vertically. What is the magnitude of acceleration for each block? Determine the tension on the cord.

Solution:

If we set a new coordinate system ($x'$, $y'$) in FBD 1 such that $x'$ is parallel to the incline, while $y'$ is perpendicular. There is no motion (acceleration, velocity etc) along $y'$. The motion on the incline is along $x'$ -direction.
If the blocks are connected and moving together; then they will have the same acceleration:

\[ a_L = a_2 = a \quad \text{(Magnitudes)} \]

\[ T - m_1 g \sin \theta = m_1 a_L \]

\[ m_2 g - T = m_2 a_2 \]

\[ T - m_1 g \sin \theta + m_2 g - T = m_1 a + m_2 a \]

\[ m_2 g - m_1 g \sin \theta = (m_1 + m_2) a \]

\[ a = \frac{m_2 g - m_1 g \sin \theta}{m_1 + m_2} \]

\[ a = \frac{(2.30 \text{ kg})(9.81 \text{ m/s}^2) - (3.70 \text{ kg})(9.81 \text{ m/s}^2) \sin 30}{(2.30 \text{ kg} + 3.70 \text{ kg})} \]

\[ a = 7.21 \text{ m/s}^2 \]
Now we can determine the tension on the cord simply by taking one of the equations from the FBDs.

\[ m_2 g - T = m_2 \alpha \implies T = m_2 (g - \alpha) \]

\[ T = (2.30 \text{ kg}) (9.81 \text{ m/s}^2 - 7.21 \text{ m/s}^2) \]

\[ T = 5.98 \text{ N} \]

**Example 3**

A block of mass \( M \) is held in balance through a system of pulleys, when a force \( F \) is applied, as shown in the figure. The pulleys are massless and frictionless. Find the tensions in each section and \( F \) in terms of \( M \).

**Solution:**

Since the system is in balance, the net force at each point of the system should be zero. Thus we need to determine specific points and draw FBDs for each.
Besides, $T_1$, $T_2$ and $T_3$ denote the tensions on the same piece of the cord. Since the cord has the same tension every point on itself

$$T_1 = T_2 = T_3 = T$$

### FBD 1

- $T_5$
- $Mg$
- $T - Mg = 0$
- $T_5 = Mg$

### FBD 2

- $T_2$
- $T_3$
- $2T - T_5 = 0$
- $T = \frac{T_5}{2}$

### FBD 3

- $T_4$
- $T_3$
- $T_4 - 3T = 0$
- $T_4 = 3T$

### FBD 4

- $T_1$
- $F$
- $T - F = 0$

\[
\begin{align*}
T_1 &= T_2 = T_3 = \frac{Mg}{2} \\
T_4 &= \frac{3Mg}{2} \\
T_5 &= Mg \\
F &= \frac{Mg}{2}
\end{align*}
\]
Example 4

Mass $m_1$ on a frictionless and horizontal table is connected to mass $m_2$ with massless and frictionless cords-pulleys. $P_1$ is allowed to move with the system, while $P_2$ is fixed.

**a)** Find the accelerations of $m_1$ and $m_2$

**b)** Determine the tensions on the cords

Express your results in terms of $m_1$, $m_2$ and the gravitational acceleration ($g$).

**Solution:**

Since $P_1$ is moving in the system, its motion leads to different accelerations for $m_1$ and $m_2$. We can see the relations between the accelerations by setting up the positions.
If cord1 is of length $L$, then from the figure

\[ x + l = L \]
\[ x_{block} = x - l \]
\[ x_{block} = x - (L - x) \]
\[ x_{block} = 2x - L \]

\[ a_{block} = \frac{d^2 x_{block}}{dt^2} = \frac{d^2}{dt^2} (2x - L) = 2 \frac{d^2 x}{dt^2} \]

\[ a_2 = \frac{d^2 x}{dt^2} \Rightarrow a_{block} = 2a_2 \]

\[ a_{block} = 2a \]
\[ a_2 = a \]

Let us draw FBDs

**FBD 1**

\[ \begin{align*}
N & \downarrow \\
\text{m}_1 & \swarrow T_1 \\
\text{m}_2 g & \downarrow \\
\end{align*} \]

N - m$_1$g = 0

**FBD 2**

\[ \begin{align*}
T_1 & \leftarrow \text{m}_1 \\
T_2 & \rightarrow \\
T_1 & \leftarrow \text{m}_2 \\
\end{align*} \]

(pulley is massless)

T$_2$ - 2T$_1$ = 0

**FBD 3**

\[ \begin{align*}
T_2 & \uparrow \\
\text{m}_2 g & \downarrow \\
\end{align*} \]

T$_2$ - m$_2$g = m$_2$ a$_2$

T$_1$ = m$_1$ a$_{block}$
Tensions on the cords can straightforwardly be calculated from the FBD equations:

\[ T_1 = m_1 (2a) , \quad T_2 = 2T_1 , \quad T_2 - m_2 g = m_2 a \]

\[ T_2 = 4m_1 a \quad 4m_1 a - m_2 g = m_2 a \]

\[ a = \frac{m_2 g}{4m_1 - m_2} \]

\[ a_{block} = 2a = \frac{2m_2 g}{4m_1 - m_2} \]

\[ a_2 - a = \frac{m_2 g}{4m_1 - m_2} \]

Accelerations of the masses

Tensions on the cords can straightforwardly be calculated from the FBD equations:

\[ T_1 = 2m_1 a \quad T_2 = 2T_1 = 4m_1 a \]

\[ T_1 = \frac{2m_1 m_2 g}{4m_1 - m_2} , \quad T_2 = \frac{4m_1 m_2 g}{4m_1 - m_2} \]
Example 5

Two blocks of masses \( m \) and \( M \) are moving under the effect of a horizontal force \( F \), as shown in the figure. The static friction coefficient between \( m \) and \( M \) is \( \mu_s = 0.38 \), while the horizontal surface is frictionless. If \( m \) is 16 kg and \( M \) is 88 kg, what should the minimum magnitude of \( F \) to keep it without sliding down?

Solution

\[
\begin{align*}
F - N &= ma \\
F - Ma &= ma \\
N - mg &= 0 \\
N - f_s - Mg &= 0
\end{align*}
\]

\[
\begin{align*}
F - N &= ma \\
N &= Ma \\
f_s - mg &= 0 \\
f_s &= Ms \Rightarrow N = Ms Ma \\
N_s - f_s - Mg &= 0 \\
Ng &= Ms Ma \\
\Rightarrow f_s &= \frac{mg}{Ms M}
\end{align*}
\]

\[
F = (M+m)a
\]
**Example 6**

A block of mass $m$ stays at rest on a horizontal table, which has static friction coefficient $\mu_s$. Then, one holds the table from one end and starts lifting up slowly. When the table make a critical angle, the block starts sliding down the table with a constant velocity. Find the critical angle in terms of the mass, static friction coefficient and the gravitational acceleration.

**Solution**

When it starts sliding, the force on the block should be equal to the static friction force between the table and the block. FBD for the block can be drawn as

$\begin{align*}
F &= (M+m)g = (M+m) \frac{mg}{M_s M} \\
F &= (88 \text{ kg} + 16 \text{ kg}) \frac{(16 \text{ kN})(9.81 \text{ m/s}^2)}{(0.38) (88 \text{ kg})} \\
F &= 488 \text{ N}
\end{align*}$
\[
\begin{align*}
N &= mg \cos(\theta_e) \\
\tau_s &= M_s mg \cos(\theta_e) \\
f_s &= mg \sin(\theta_e)
\end{align*}
\]

\[
\begin{align*}
M_s mg \cos(\theta_e) &= mg \sin(\theta_e) \\
M_s &= \frac{\sin(\theta_e)}{\cos(\theta_e)} \\
\tan \theta_e &= M_s \\
\theta_e &= \arctan(M_s)
\end{align*}
\]